FORM 3TBP

The College Board
Advanced Placement Examination
CALCULUS AB
SECTION II

This green insert may be used for reference and/or scratchwork as you answer the free-response questions, but be sure to show all your work and your answers in the pink booklet. No credit will be given for work shown on this green insert.

CALCULUS AB

SECTION II

Time — 1 hour and 30 minutes

Number of problems — 6

Percent of total grade — 50

A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR PARTS OF PROBLEMS ON THIS SECTION OF THE EXAMINATION.

REMEMBER TO SHOW YOUR SETUPS AS DESCRIBED IN THE GENERAL INSTRUCTIONS.

General instructions for this section are printed on the back cover of the test booklet.

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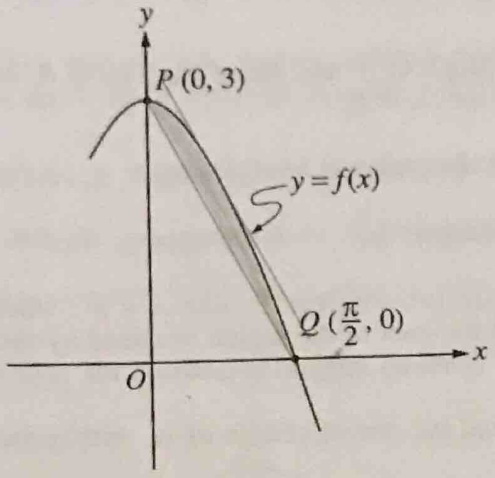
1. A particle moves along the x-axis so that its velocity at any time $t \geq 0$ is given by $v(t) = 3t^2 - 2t - 1$. The position $x(t)$ is 5 for $t = 2$.

   (a) Write a polynomial expression for the position of the particle at any time $t \geq 0$.

   (b) For what values of t, $0 \leq t \leq 3$, is the particle's instantaneous velocity the same as its average velocity on the closed interval $[0, 3]$?

   (c) Find the total distance traveled by the particle from time $t = 0$ until time $t = 3$.



2. Let f be the function given by $f(x) = 3\cos x$. As shown above, the graph of f crosses the y-axis at point P and the x-axis at point Q.

(a) Write an equation for the line passing through points P and Q.

(b) Write an equation for the line tangent to the graph of f at point Q. Show the analysis that leads to your equation.

(c) Find the x-coordinate of the point on the graph of f, between points P and Q, at which the line tangent to the graph of f is parallel to line PQ.

(d) Let R be the region in the first quadrant bounded by the graph of f and line segment PQ. Write an integral expression for the volume of the solid generated by revolving the region R about the x-axis. Do not evaluate.

$f'(c) = \frac{f(b) - f(a)}{b - a}$

$-3\sin c = \frac{0 - 3}{\frac{\pi}{2} - 0}$

$-3\sin c = -3 \div \frac{\pi}{2}$

$-\frac{3\pi}{2}\sin c = -3$

$\sin c = -3 \cdot -\frac{2}{3\pi}$

$= \frac{6}{3\pi} = \frac{2}{\pi}$

$\sin c = \frac{2}{\pi}$

$c = \sin^{-1}\left(\frac{2}{\pi}\right)$

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3. Let f be the function given by $f(x) = \sqrt{x - 3}$. AB

(a) On the axes provided below, sketch the graph of f and shade the region R enclosed by the graph of f, the x-axis, and the vertical line $x = 6$.

Note: The axes for this graph are provided in the pink test booklet only.

(b) Find the area of the region R described in part (a).

(c) Rather than using the line $x = 6$ as in part (a), consider the line $x = w$, where w can be any number greater than 3. Let $A(w)$ be the area of the region enclosed by the graph of f, the x-axis, and the vertical line $x = w$. Write an integral expression for $A(w)$.

(d) Let $A(w)$ be as described in part (c). Find the rate of change of A with respect to w when $w = 6$.

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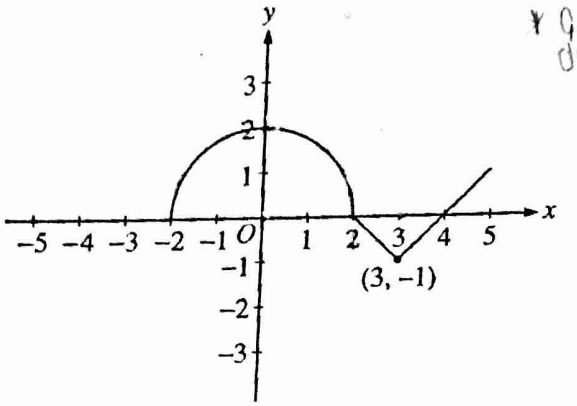
4. Let f be the function given by $f(x) = x^3 - 6x^2 + p$, where p is an arbitrary constant. AB

(a) Write an expression for $f'(x)$ and use it to find the relative maximum and minimum values of f in terms of p. Show the analysis that leads to your conclusion.

(b) For what values of the constant p does f have 3 distinct real roots?

(c) Find the value of p such that the average value of f over the closed interval $[-1, 2]$ is 1.



* g''(2) cannot exist if f'(2) doesn't exist at that point

5. The graph of a function f consists of a semicircle and two line segments as shown above. Let g be the function given by $g(x) = \int_0^x f(t)\,dt$.

(a) Find $g(3)$.

(b) Find all values of x on the open interval $(-2, 5)$ at which g has a relative maximum. Justify your answer.

(c) Write an equation for the line tangent to the graph of g at $x = 3$.

(d) Find the x-coordinate of each point of inflection of the graph of g on the open interval $(-2, 5)$. Justify your answer.

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